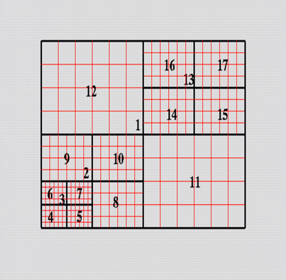
Using adaptive structured meshes for MHD equations

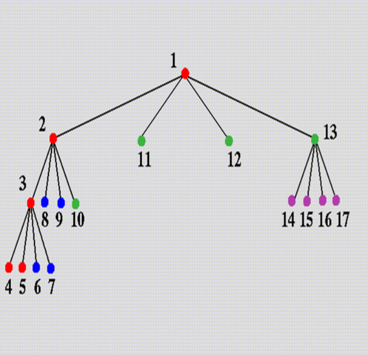
Guillaume LATU (LSIIT-Strasbourg & INRIA Sud-Ouest)

References : Talk of D. Schnack, SCIDAC

AMR meshes

- smooth zones : coarse grid
- unsmooth zones : fine grid



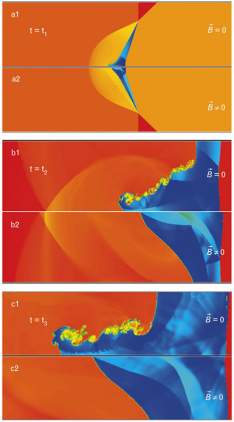


AMR Solvers

- Specific numerical components :
  - refinement/coarsening criterion
    acts on accuracy and convergence rate
    could depend on gradient of functions, vorticity
  - numerical correction at coarse/fine block interface

- Explicit :
  - Time Subcycling is performed to relax the CFL criterion that links *dt* and *dx*
    $\longrightarrow$ Many subcycling iteration on small patches
  - AMR ties temporal resolution to patch size

- Implicit :
  - AMR helps to find an adaptive mesh
  - Big linear system to solve



MHD equations

- Numerical integration
  - Small-scale spatial structures develop
    Adaptive mesh refinement (AMR) provides high resolution
  - Extended MHD induces dispersive waves
    Sharp CFL condition for explicit codes (typically $\Delta t \propto \Delta x^2$)
    Implicit time stepping is useful
- Implicit solving :
  - Efficient preconditioners could be hard to find
  - Direct solver to handle the ill-conditioned systems

Advanced numerical schemes

- Partially-implicit schemes
  - Treat fastest waves implicitly
  - Slow waves and transport solved with explicit scheme
  - Time step still limited by slow waves
- Semi-implicit
  - All waves treated implicitly
  - Time step limited by transport
- Partially-implicit and Semi-implicit schemes
  Simplification & splitting $\longrightarrow$ potentially inaccurate
- Fully Implicit
  - Arbitrary time step
  - Non linear coupling in equations, anisotropy
  - Ill-conditioned matrices (specialized pre-conditioners)
  - $\longrightarrow$ Problem of scalability on large platform

Geometry

- Toroidal direction
  Long wavelengths, periodicity => FFT or finite diff.
- Poloidal plane
  Fine structure across field direction
  Grids aligned with flux surfaces (~field lines)
  Unstructured triangular grids
  Extreme packing in specific areas
- Finite elements
  High order elements essential for resolving anisotropies

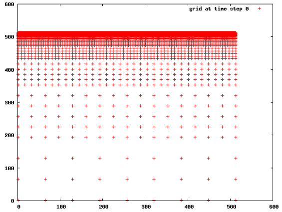
AMR & MHD

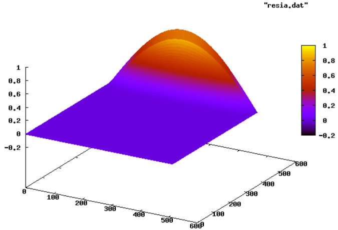
- AMR uses :
  or block-based refinement (usually 1 block size)
  or cell-based refinement (unfrequent)
- AMR could help defining zones for
  explicit time integration
  parallelization
- AMR & implicit scheme : difficult to manage parallel sparse matrice sparse structure are dynamic (overhead) parallel preconditioner needed (often)

PARAMESH

- package of Fortran 90 subroutines (uses MPI library).
- logically cartesian structured mesh is automatically managed by a parallel code with adaptive mesh refinement (AMR).
- builds a hierarchy of sub-grids to cover the computational domain, with spatial resolution varying
- sub-grid blocks form the nodes of a tree data-structure (quad-tree in 2D or oct-tree in 3D). Each grid block has a logically cartesian mesh.
- operates as a parallel domain decomposition tool.
- `SAMRAI`, `Grace` are an alternative choice to AMR calculation.

Heat equation on adaptive grid





- Pb to evaluate accurately derivative, laplacian on adpative grid
- Efficient implementation needed to compete with non-adaptive code (memory & computation costs)
- Very sparse system could give efficient code