Numerical simulation of tokamak plasmas

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We are interested in the numerical simulation of of magneto-hydro-dynamic instabilities in tokamak plasmas [1]. In this study, two codes are developed and compared : JOREK (Euratom and CEA) and FluidBox (INRIA). We also investigate a solver based on adaptive grid and implicit scheme.

The main application of JOREK [3] is a specific type of MHD instability named Edge Localised Mode or ELM. This instability occurs at the edge of the tokamak plasma, specifically in a magnetic geometry with an x-point. The ELMs are of concern for the ITER experiment due to the large peak heat losses induced by this instability. These losses may induce damage to the wall of the machine.

The MHD equations solved in the JOREK code is the reduced MHD model in toroidal geometry. The reduction consists in a potential representation of both the flow and the magnetic field.

For the simulation of ELMs, the simulation domain must include both the closed magnetic field lines in the centre of the plasma and the open field lines outside. For numerical accuracy it is important that the variables are aligned with the initial magnetic flux surfaces. In the JOREK code finite elements are used in the two non-periodic dimensions, Fourier harmonics are used in the periodic, toroidal, direction. The present version of the JOREK code uses iso-parametric cubic Hermite finite elements. Thus both the two space coordinates and all the variables are represented by the same finite elements. This allows an accurate adaptation of the grid to the magnetic geometry.

The first applications are the poloidal flows induced in the equilibrium with an x-point and the simulation of so-called ballooning modes, the instability underlying the ELMs.

The code FluidBox is a more general purpose CFD code but is less advanced for MHD problems. It uses a different numerical method : triangular type meshes and a stabilised residual distribution method [1]that is able to handle strong gradients. The $\operatorname{div} B = 0$ condition is handled thanks to a Lagrange multiplicator technique. The code is implicit, fully parallel and uses the parallel solver PaStiX [4]. An example of simulation is given on Figure 1. This example is taken from [2].

In addition, in the framework of adaptive grids, the system solved at each time step could be decomposed into two parts : the first one corresponds to the coarsest level of refinement and we compute an exact inverse of this system by a direct factorization ; the second one corresponds to the unknwons added by the refinement ; the solution on this part of the system is obtained using an iterative method on the Schur complement system. To validate this approach, we design a prototype using hierarchical finite element basis.

During the conference we will describes the numerical schemes and strategy, present results obtained by the two codes on the ELM instability.

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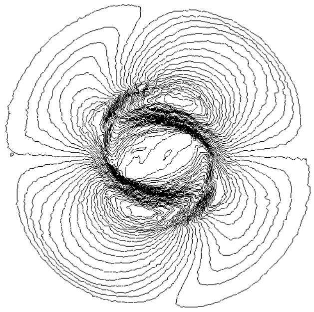

FIG. 1 – Magnetic pressure for a rotor case. The initial field is such that $||\vec{u}|| = 2$ and is clockwise oriented from the origin to a radius of 0.1, then anti clockwise to $r = 0.115$ and then is zero. The magnetic field is uniform $(B_x, B_y) = (0, 5)$, the pressure is uniform $p = 1$ the density is 10 and then decreases linearly after $r = 0.115$ to reach $\rho = 1$.

Références

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