# Implementing the full MHD equations in JOREK

project ASTER,
financé par ANR, CEA collaboration with
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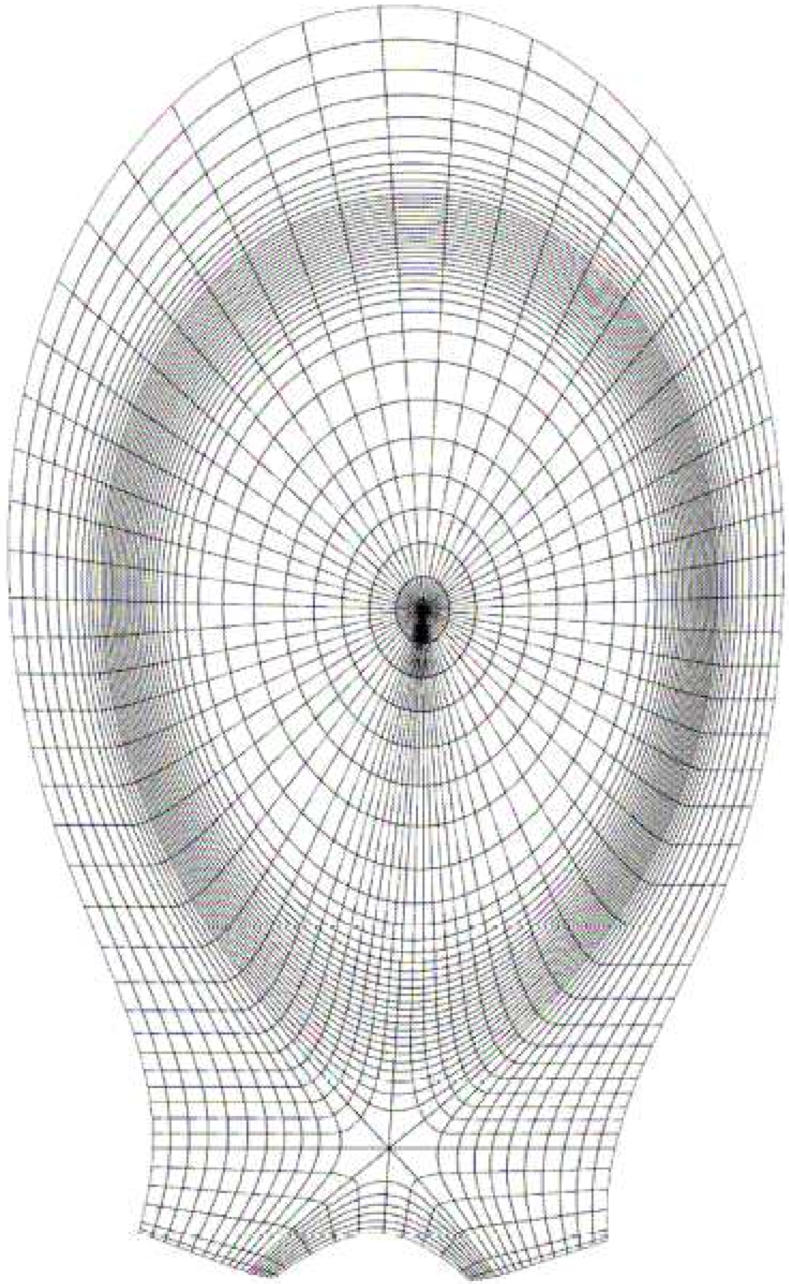
Financé par ANR

# Outline

- Introduction
- JOREK 2.0: RMHD
- Development: MHD equations
- Bezier elements

# JOREK 2.0

- At present:
  - Reduced MHD model
  - Cubic Bezier finite elements
    - 2-D elements & Fourier harmonics
    - X-point
    - open/closed field lines
  - fully implicit (Crank-Nicholson) scheme
    - allows large time steps



# Reduced MHD

- The reduced MHD equations: scalar

$$\partial_t \rho = -(1+\epsilon x)[\phi,\rho] + 2\epsilon\rho\partial_y\phi + \nabla\cdot(D_\perp\nabla_\perp\rho) + S_\rho,$$

$$\partial_t w = -(1+\epsilon x)[\phi,w] + 2\epsilon\partial_y\phi + \frac{1}{1+\epsilon x}[\psi,J] - \frac{\epsilon}{(1+\epsilon x)^2}\partial_3 J + \nu\nabla_\perp^2 w,$$

$$\rho\partial_t T = -(1+\epsilon x)\rho[\phi,T] + 2\epsilon\rho T\partial_y\phi + \nabla\cdot(\mathsf{K}\nabla T) + S_T,$$

$$\partial_t\psi = -(1+\epsilon x)[\phi,\psi] + \eta\Delta^*\psi - \epsilon\partial_3\phi.$$

- Uses magnetic vector potential ψ to ensure $\nabla\cdot\mathbf{B} = 0$
- variables: $[\psi,\phi,w,\rho,T,j]^T$

- Used to study ELM physics

# JOREK development : full MHD

## ⇨ comparison with experiment: MHD

- Introduces new physics
  - ⇨ compressibility, magneto-acoustic waves
  - ⇨ effects of parallel flow
  - ⇨ parallel vs perpendicular physics untangled

The full MHD equations:

$$\partial_t \rho = -\nabla \cdot (\rho \mathbf{v}) + \nabla \cdot (D \nabla \rho) + S_\rho,$$

$$\rho \partial_t \mathbf{v} = -\rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla(\rho T) + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{v},$$

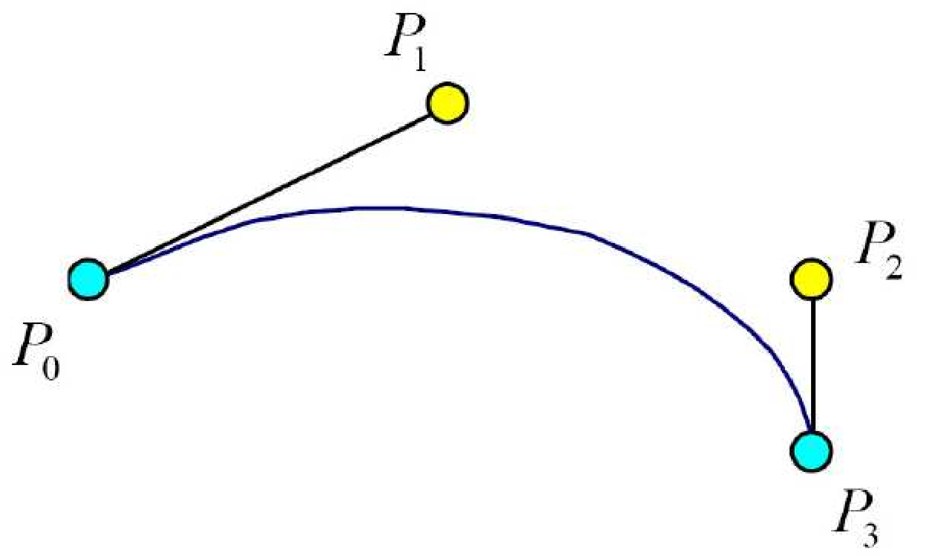
$$\partial_t T = -\mathbf{v} \cdot \nabla T + (\gamma - 1) T \nabla \cdot \mathbf{v} + \nabla \cdot (\bar{\mathsf{K}} \nabla T) + S_T,$$

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}).$$

- – Vector equations: $[\rho, v^1, v^2, v^3, T, A_1, A_2, A_3]^T$
- – needs projection onto vector basis

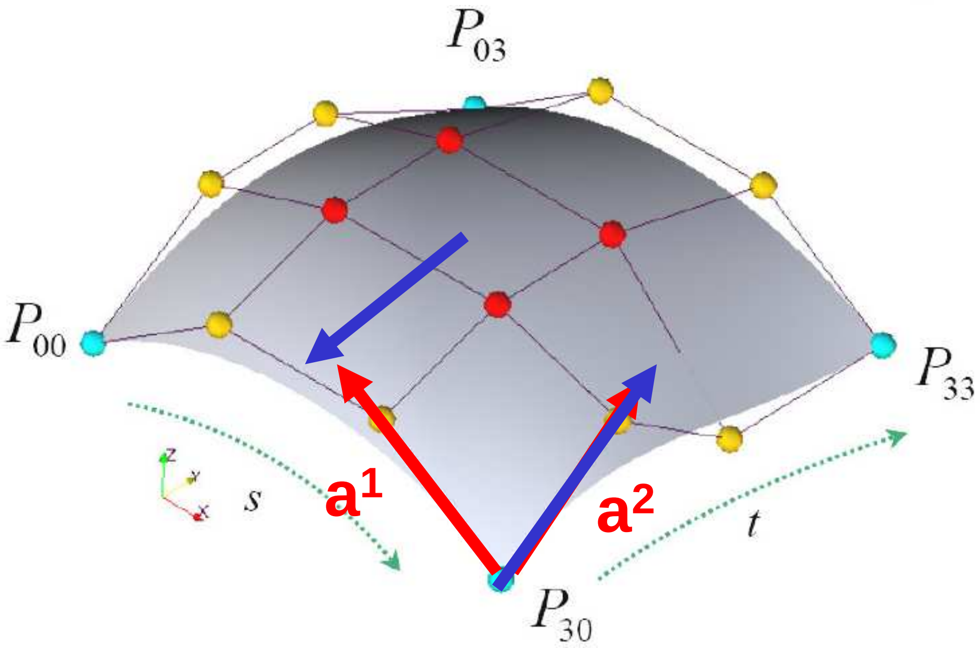
# Bezier finite elements

- 1D Bezier curves
  - 2 vectors
  - 4 points




- 2D Bezier patches
  - 16 points, or
  - 3 vectors per node
- continuity up to first order

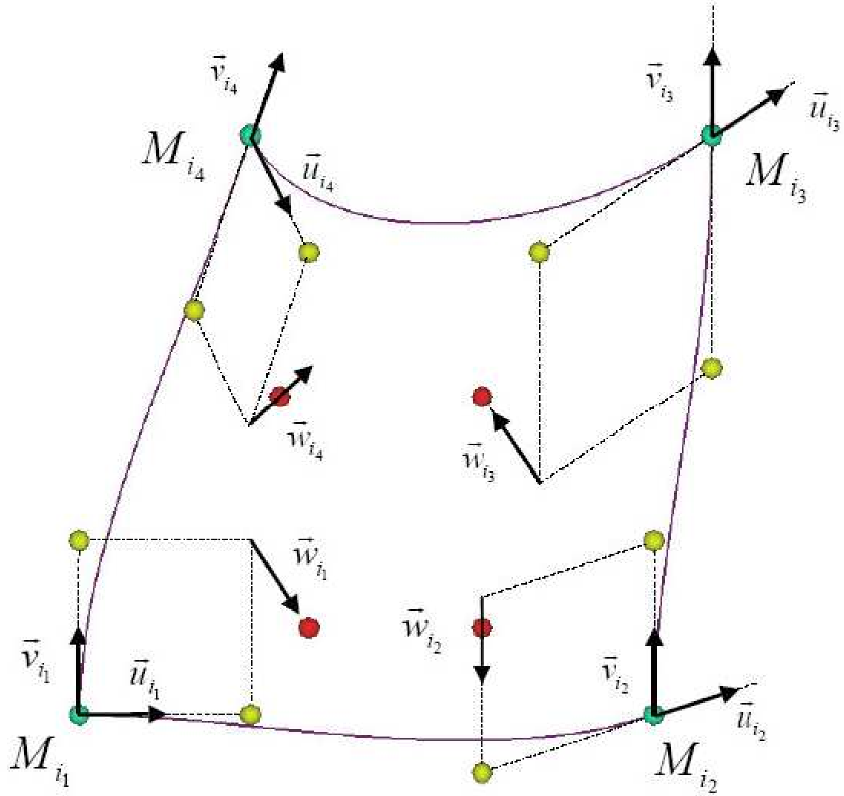
- flexible discretisation
- easily refinable: AMR
- basis for vector formalism



# Reformulation of MHD equations

- Bezier grid determines spatial discretisation and vector basis
  - flux aligned, also at separatrix
  - non-orthogonal curvilinear coordinate system
- Geometry defines the problem!

  Requires thorough consideration of the underlying metric



# Metric

- with the local coordinates {s,t,ϕ} we obtain a metric:

$$\mathbf{a}_1 = \mathcal{J}(\nabla t \times \nabla\phi), \qquad \mathbf{a}^1 = \nabla s,$$
$$\mathbf{a}_2 = \mathcal{J}(\nabla\phi \times \nabla s), \qquad \mathbf{a}^2 = \nabla t,$$
$$\mathbf{a}_3 = \mathcal{J}(\nabla s \times \nabla t), \qquad \mathbf{a}^3 = \nabla\phi,$$

- with Jacobian $\mathcal{J} \equiv \nabla s \times \nabla t \cdot \nabla\phi$

- These are related to grid quantities by

$$\begin{pmatrix} s_x & t_x \\ s_y & t_y \end{pmatrix} = \frac{1}{x_s y_t - x_t y_s} \begin{pmatrix} y_t & -y_s \\ -x_t & x_s \end{pmatrix}$$

# What are demands on this basis?

- Finite element formulation using Galerkin weak form:
  e.g. momentum:

$$\int \mathrm{d}^3V \rho \mathbf{v}^* \partial_t \mathbf{v} = \int \mathrm{d}^3V - \rho \mathbf{v}^* \cdot (\mathbf{v} \cdot \nabla)\mathbf{v} - (\rho T + \frac{1}{2}B^2)\nabla \cdot \mathbf{v}^* + \mathbf{v}^* \cdot (\mathbf{B} \cdot \nabla)\mathbf{B}$$

- Choose projection: co / contra variant:
- gradient: $\nabla \equiv \mathbf{a}^1 \partial_1 + \mathbf{a}^2 \partial_2 + \mathbf{a}^3 \partial_3$
- velocity $\mathbf{v} = v^1 \mathbf{a}_1 + v^2 \mathbf{a}_2 + v^3 \mathbf{a}_3$
- vector potential $\mathbf{A} = A_1 \mathbf{a}^1 + A_2 \mathbf{a}^2 + A_3 \mathbf{a}^3$
  and test functions oppositely.

- Example:

$$((\mathbf{B} \cdot \nabla)\mathbf{B})^1 = B^i(\partial_i B^1) + B^i B^j (\Gamma_{ij}^k \mathbf{a}_k)^1 = B^i(\partial_i B^1) + B^i B^j \Gamma_{ij}^1$$

$$\partial_i \mathbf{a}_j \equiv \Gamma_{ij}^k \mathbf{a}_k$$

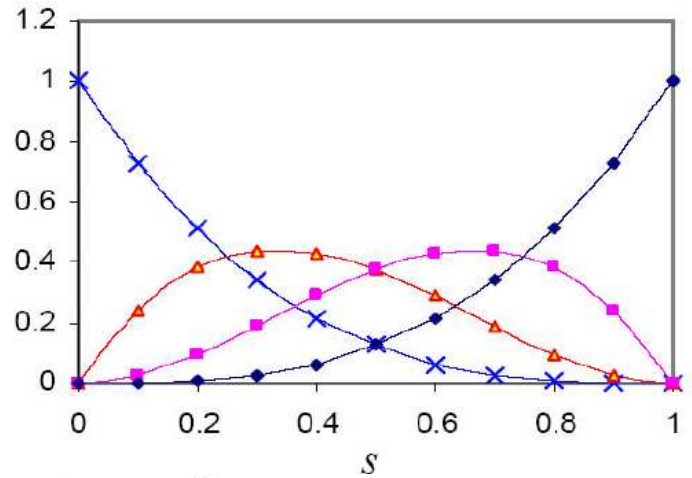
- 2nd derivative of local coordinate

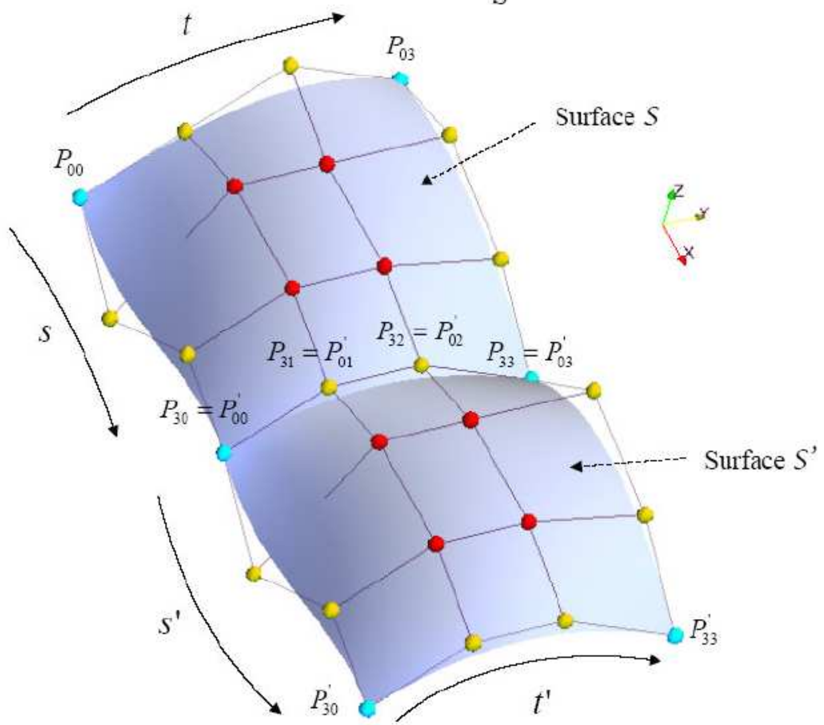
$$\Gamma_{ij}^k \equiv \frac{1}{2} g^{mk} (\partial_i g_{mj} + \partial_j g_{im} - \partial_m g_{ij})$$

# Continuity of Cubic Bezier elements

- Cubic Bezier elements match value and first derivative

- 2nd derivative discontinuous (e.g. $x_{ss}$, $x_{tt}$, not $x_{st}$ !)




- geometric connections (christoffel, jacobian)
- magnetic potential (current)
- viscosity (?)

⇨ grid resolution
⇨ reformulation of equations?

# Conclusions

- Using local coordinates of cubic Bezier grid to define vector basis for MHD equations

  ⇨ project variables onto vector basis

- JOREK: implementation under development

- Next:
  - testing code
  - Adaptive Mesh Refinement

# To do

First year

- Implementation of MHD equations, testing
- Implementation of Adaptive Mesh Refinement

Second year

- Simulate full ELM cycle!
- Then: code available to
  - study influence of plasma parameters on ELM size
  - study ELM relaxation mechanism
  - study influence of Resonant Magnetic Perturbations
  - ...

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