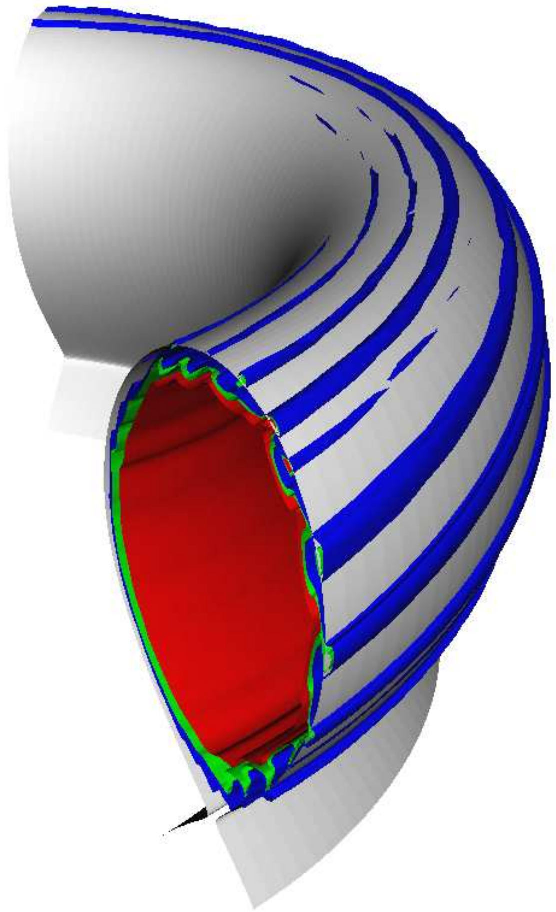
Adaptive mhd Simulation of Tokamak Elms for iteR

Projet ANR-CIS 2006

Meeting Bordeaux

29-30/5/2008



WP 4.1

- Application of the methods developed in the project to advance the simulation of ELMs in ITER plasmas
  - Implementation of parallel velocity in existing JOREK reduced MHD model:

$$\vec{v} = -R\vec{\nabla}u(t)\times\vec{e}_\varphi + v_\parallel(t)\vec{B} \qquad \vec{B} = \frac{F_0}{R}\vec{e}_\varphi + \frac{1}{R}\vec{\nabla}\psi(t)\times\vec{e}_\varphi$$

$$\vec{B}\cdot\left(\rho\frac{\partial\vec{v}}{\partial\tilde{t}} = -\rho\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} - \vec{\nabla}(\rho T) + \vec{J}\times\vec{B} + \mu\Delta\vec{v}\right)$$

  - Boundary condition:

$$v_\parallel = c_s = \sqrt{\gamma T_{divertor}}$$

•Simple condition leads to inflow of particles due to large poloidal flow induced by the MHD instability

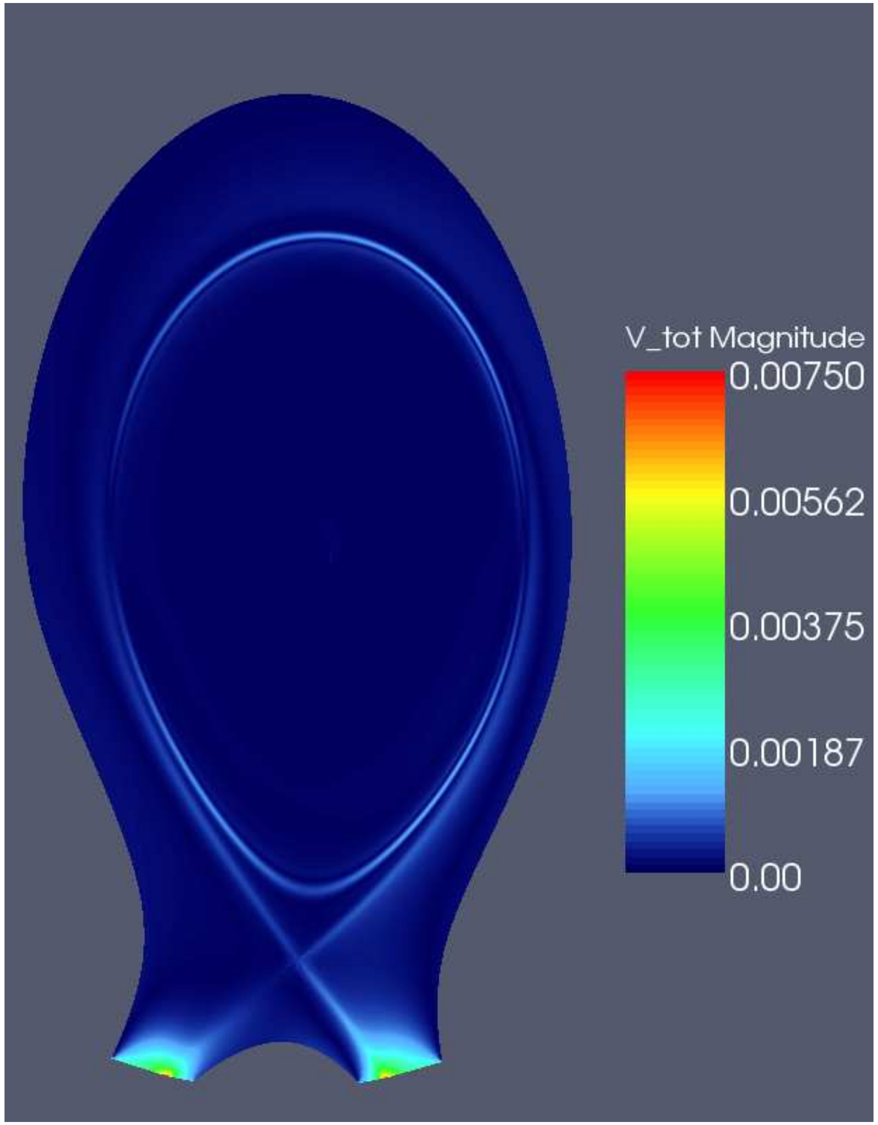
•Generalisation of Mach-1 condition to include perpendicular flow component

$$\vec{v}_\parallel\cdot\vec{n} + \vec{v}_E\cdot\vec{n} = \frac{\vec{v}_\parallel\cdot\vec{n}}{\left|v_\parallel\right|}c_s$$

•Free outflow density and temperature (to be updated to sheath transmission factors)

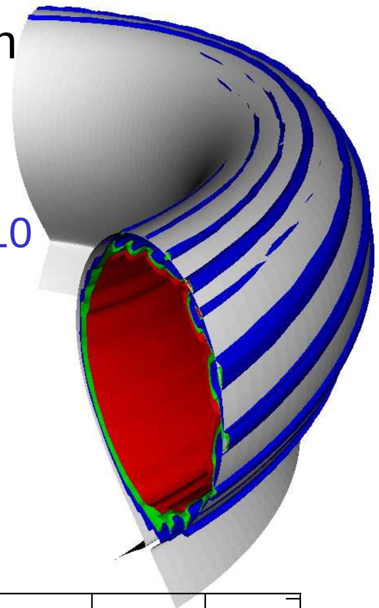
WP4.1 Equilibrium Flow

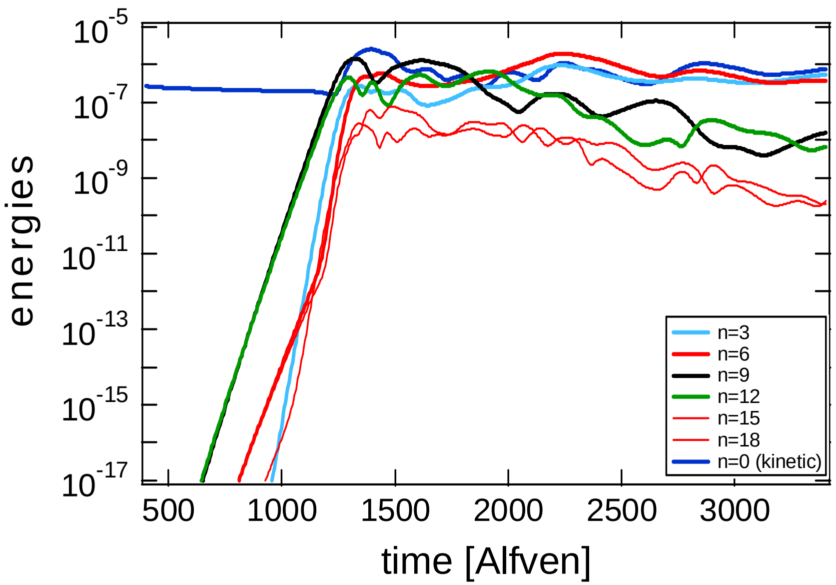
- Equilibrium flow:
  - Parallel component at the divertor target
  - Poloidal component due to edge pressure gradient and diffusion/resistivity terms

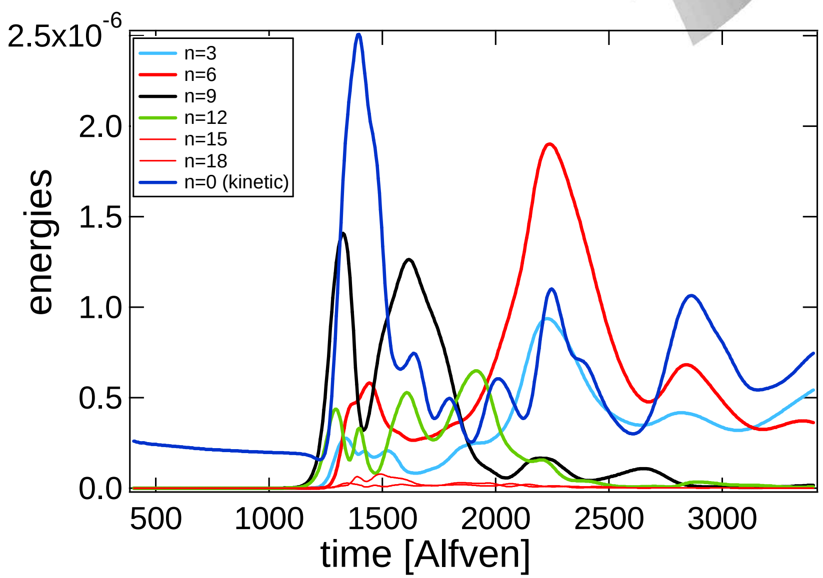


First ELM simulation using new JOREK

- Evolution of a plasma unstable to medium-n ballooning mode(s) in the edge region
  - n=0-18, periodicity 3
  - Resistivity $2x10^{-6}$, viscosity $4x10^{-6}$, Diffussivity 10^{-5} conductivity $10^{-5}/10$
- bursts of n=9 ballooning, later n=6 is destabilised
  - Large equilibrium flow induced by the instability
  - formation of density filaments on outboard side

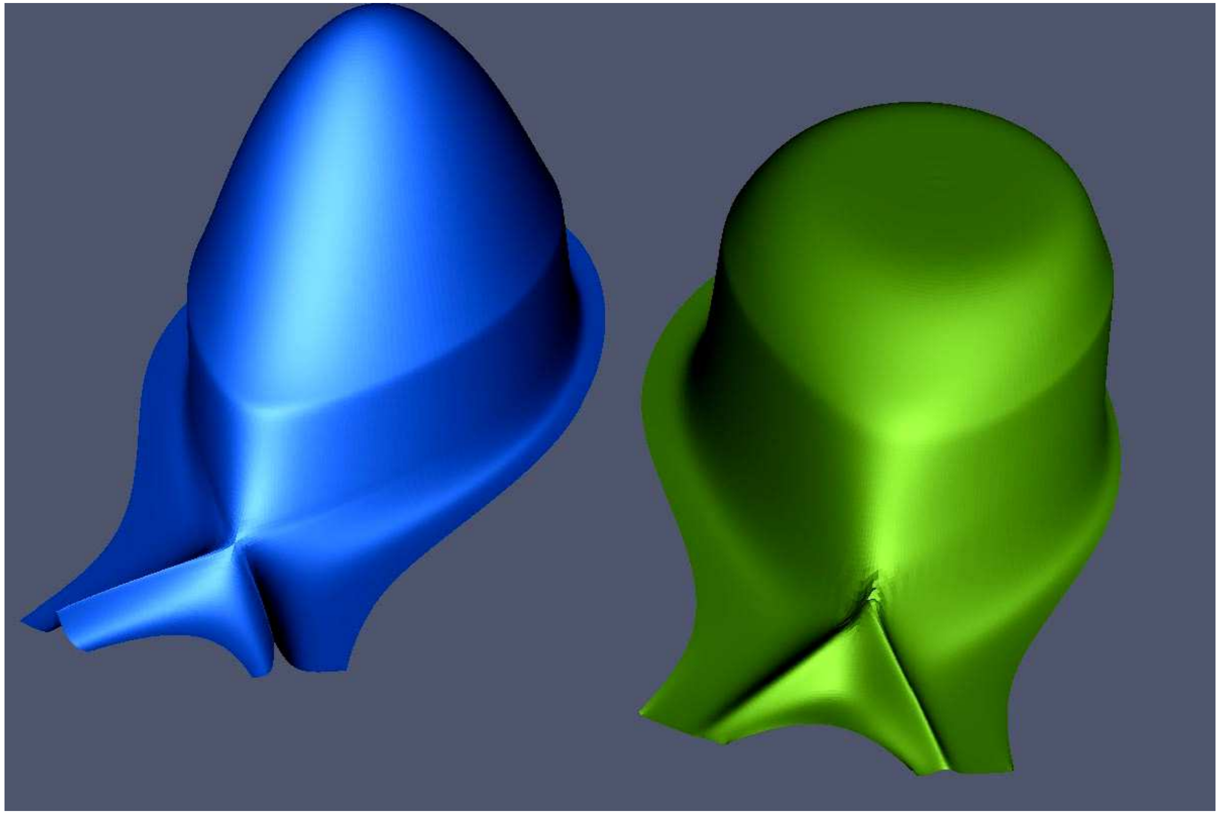






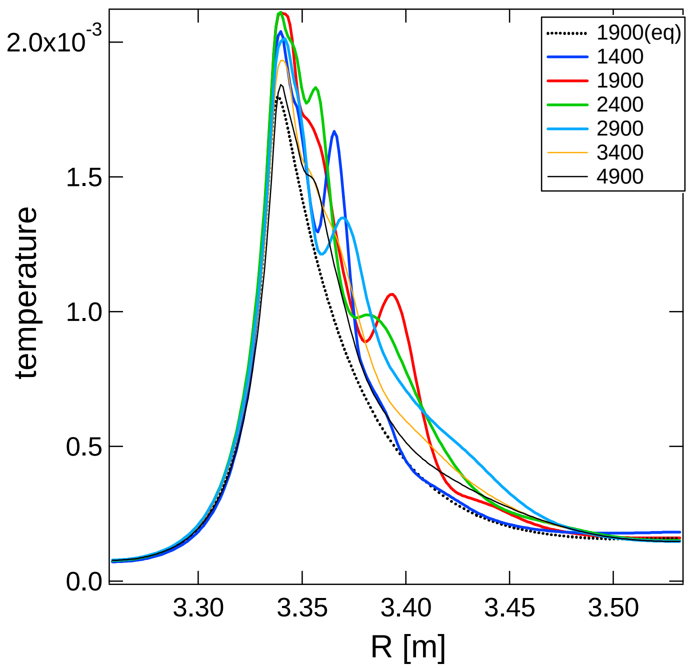
Profile Evolution

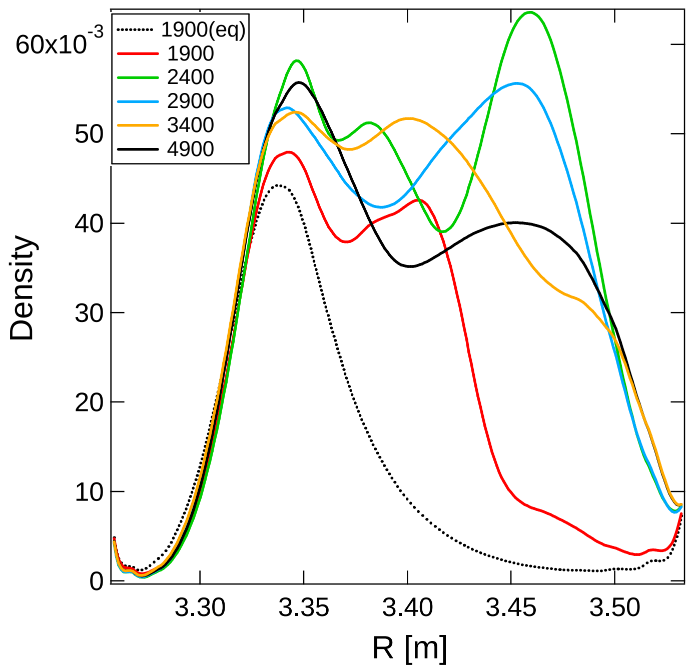
- Evolution temperature and density profiles:

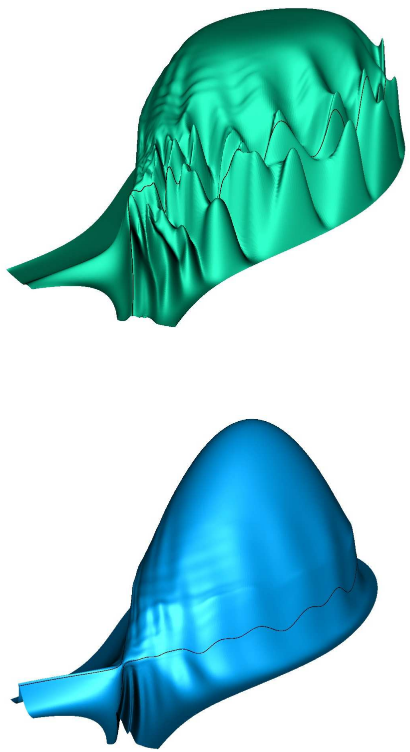


Divertor Target Profiles

- First results on energy and density losses to the divertor target:
  - Fine structure on temperature profile (due to magnetic perturbation)
  - Slower broader density perturbation due to filaments arriving in divertor







Planning 2008

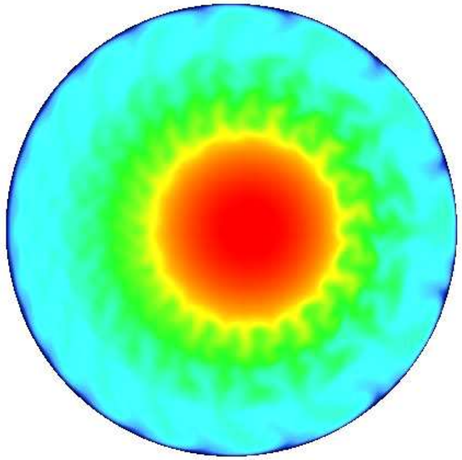
- JOREK2 developments (WP2):
  - Include PastiX with distributed matrix, test direct/indirect version
  - Change MHD model to full MHD, diamagnetic effects
  - Include refinement in new JOREK version
  - Post-doc Emiel van der Plas started 1-2-2008

- JOREK applications (WP4)
  - Equilibrium flow patterns due to X-point configuration
    - almost complete (S. Pamela, abstract Varenna Fusion Theory Conference)
  - Extend ELM simulations to higher toroidal resolution
    - Saturation mechanism
    - Crash amplitude

‘turbulence’

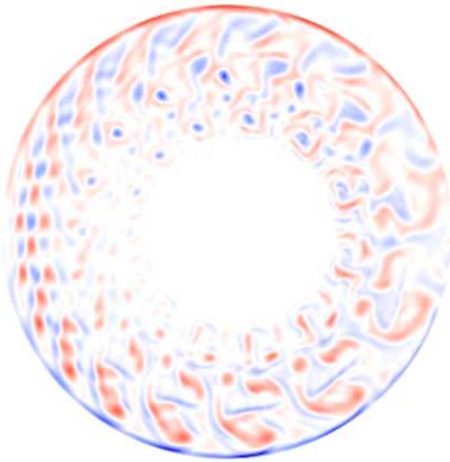
- First tests of possibility to do turbulence with JOREK
  - Resistive ballooning turbulence (n=0-48,6), (n_r=41, n_p=128)

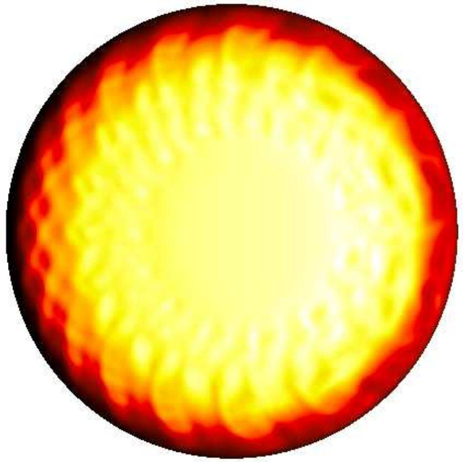
density



temperature

vorticity





current density

JOREK : real or complex

- JOREK version using complex variables

$$u(\varphi) = \sum u_n e^{in\varphi} + u_n^* e^{-in\varphi}$$

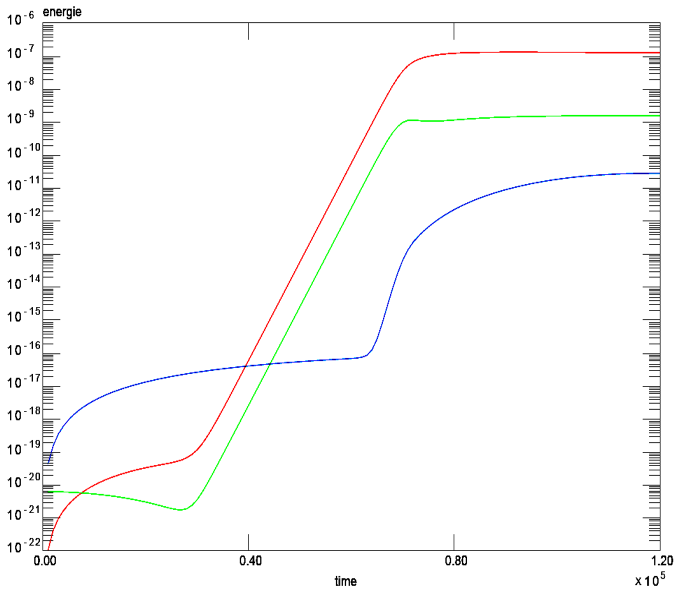
  – Tearing mode test cases diverges after 10^5 Alfven times (in steady state with island)
  – Ballooning mode cases diverge in the early non-linear phase

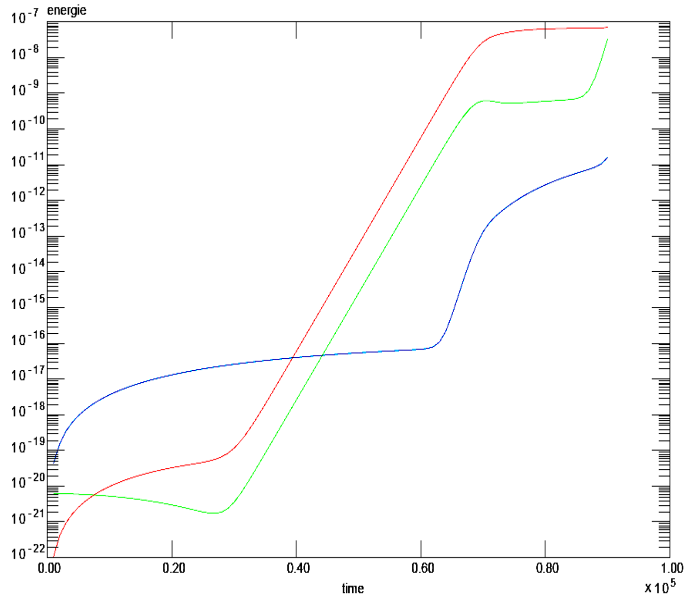
- using real variables

$$u(\varphi) = \sum u_{n,c} \cos(n\varphi) + u_{n,s} \sin(n\varphi)$$

  – Stable for both tearing and ballooning mode cases
  – But slower for matrix construction/solution, larger memory requirements

The Problem


energie
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
0.00
0.40
0.80
1.20
time
$x\,10^{5}$


energie
10^{-7}
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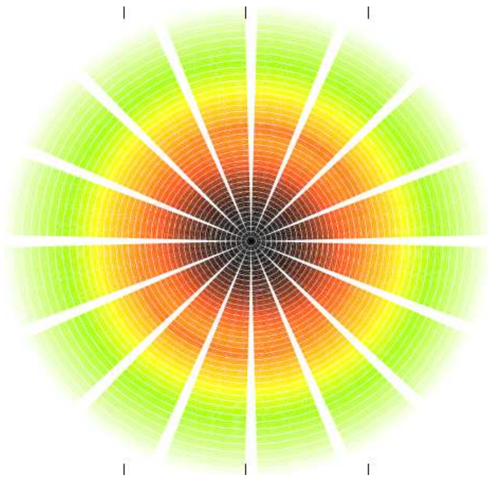
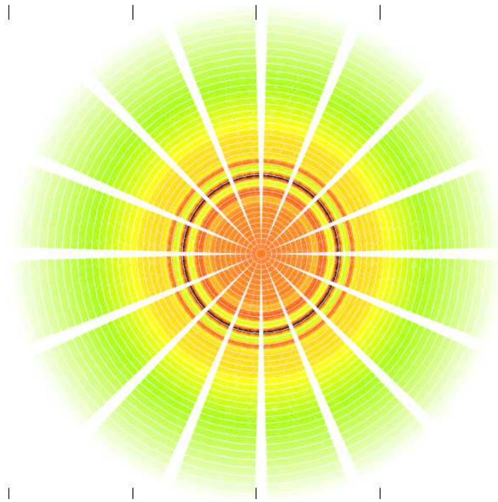
0.80
1.00
time
$x\,10^{5}$





Implicit n=0 equation

- Implicit complex scheme cannot be reduced to remove conjugate harmonics from the equation for n=0:

$$\frac{\partial\psi}{\partial t}=F(\psi)$$

$$\delta\psi=\partial t\left(F+\tfrac{1}{2}F'\delta\psi\right)$$

$$\left(1-\tfrac{1}{2}\delta tF'\right)\delta\psi=\delta tF$$

$$\psi(\varphi)=\mathrm{Re}\left(\psi(\varphi)\right)=\mathrm{Re}\left(\psi_0+\psi_n e^{in\varphi}+\psi_n^* e^{-in\varphi}\right)=\mathrm{Re}\left(\psi_0\right)+2\,\mathrm{Re}\left(\psi_n e^{in\varphi}\right)$$

$$F'\delta\psi=\left(f_0+f_n e^{+in\varphi}+f_n^* e^{-in\varphi}\right)\left(\delta\psi_0+\delta\psi_n e^{+in\varphi}+\delta\psi_n^* e^{-in\varphi}\right)$$
$$=\left(f_0\delta\psi_0+f_n\delta\psi_n^*+f_n^*\delta\psi_n\right)+\left(f_0\delta\psi_n+f_n\delta\psi_0\right)e^{+in\varphi}+\left(f_0\delta\psi_n^*+f_n^*\delta\psi_0\right)e^{-in\varphi}$$

$$\begin{pmatrix}1-\tfrac{1}{2}\delta tf_0 & -\tfrac{1}{2}\delta tf_n^* & -\tfrac{1}{2}\delta tf_n\\ -\tfrac{1}{2}\delta tf_n & 1-\tfrac{1}{2}\delta tf_0 & 0\\ -\tfrac{1}{2}\delta tf_n^* & 0 & 1-\tfrac{1}{2}\delta tf_0\end{pmatrix}\begin{pmatrix}\delta\psi_0\\ \delta\psi_n\\ \delta\psi_n^*\end{pmatrix}=\begin{pmatrix}\delta tF_0\\ \delta tF_n\\ \delta tF_n^*\end{pmatrix}$$

- Implicit real scheme keeps all terms:

$$\psi(\varphi)=\psi_0+\psi_c\cos(\varphi)+\psi_c\sin(\varphi)$$

$$F'\delta\psi=\left(f_0+f_c\cos(\varphi)+f_s\sin(\varphi)\right)\left(\delta\psi_0+\delta\psi_c\cos(\varphi)+\delta\psi_c\sin(\varphi)\right)$$

$$\begin{pmatrix}\left(1-\tfrac{1}{2}\delta tf_0\right) & -\tfrac{1}{4}\delta tf_c & -\tfrac{1}{4}\delta tf_s\\ -\tfrac{1}{4}\delta tf_c & \tfrac{1}{2}\left(1-\tfrac{1}{2}\delta tf_0\right) & 0\\ -\tfrac{1}{4}\delta tf_s & 0 & \tfrac{1}{2}\left(1-\tfrac{1}{2}\delta tf_0\right)\end{pmatrix}\begin{pmatrix}\delta\psi_0\\ \delta\psi_c\\ \delta\psi_s\end{pmatrix}=\begin{pmatrix}\delta tF_0\\ \tfrac{1}{2}\delta tF_c\\ \tfrac{1}{2}\delta tF_s\end{pmatrix}$$

  – Insert complex variables:

$$\psi_n=\tfrac{1}{2}\psi_c-\tfrac{1}{2}i\psi_s$$
$$\psi_n^*=\tfrac{1}{2}\psi_c+\tfrac{1}{2}i\psi_s$$
$$\psi_c=\left(\psi_n+\psi_n^*\right)$$
$$\psi_s=i\left(\psi_n-\psi_n^*\right)$$

$$\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_0-\tfrac{1}{2}\delta t\left(f_n\delta\psi_n^*+f_n^*\delta\psi_n\right)=\delta tF_0$$
$$-\tfrac{1}{2}\delta tf_n\delta\psi_0+\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_n=\tfrac{1}{2}\delta tF_n$$
$$-\tfrac{1}{2}\delta tf_n^*\delta\psi_0+\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_n^*=\tfrac{1}{2}\delta tF_n^*$$

  – but conjugate variables not available!

Options

- Option 1: correct RHS with explicit term from previous time step:

$$\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_0-\tfrac{1}{2}\delta tf_n^{*}\delta\psi_n=\delta tF_0+\tfrac{1}{2}\delta tf_n\delta\psi_n^{*}=\delta tF_0+\tfrac{1}{2}\left(\delta tf_n^{*}\delta\psi_n\right)^{*}$$

$$-\tfrac{1}{2}\delta tf_n\delta\psi_0+\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_n=\tfrac{1}{2}\delta tF_n$$

  – works fine for tearing mode case but not for ballooning mode

- Alternative, to keep all real contributions implicit:

$$\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_0-\delta tf_n^{*}\delta\psi_n=\delta tF_0-i\,\mathrm{Im}\left(\delta tf_n^{*}\delta\psi_n\right)$$

$$-\tfrac{1}{2}\delta tf_n\delta\psi_0+\left(1-\tfrac{1}{2}\delta tf_0\right)\delta\psi_n=\tfrac{1}{2}\delta tF_n$$

  – works fine for tearing mode case but not for ballooning mode

- Other ideas?
  – Keep to real version for the moment

• Complex version with n=0 correction: