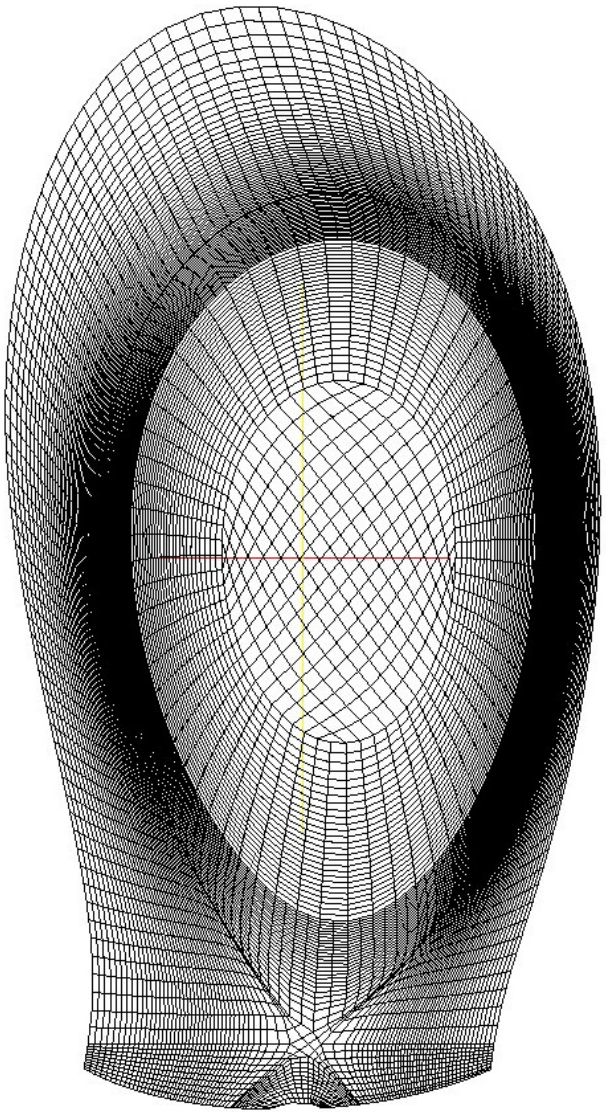
WP 2.2 : Development of the JOREK MHD simulation code

- Status JOREK-1.0, numerics:
  - Reduced MHD model
    (compare Euler vs. Navier-Stokes)
  - Generalised finite elements
    (only 2D linear elements + Fourier used)
  - Fully implicit time evolution
    (linearised Crank-Nicholson)
    - Limits size of problems to very few toroidal harmonics:
      on local system (32 CPU, 64 GB) : n=0+6, 28.000 FE, ~500.000 dof
      (limit due to memory, CPU/time step ~3 min.)



- Physics:
  - First results presented and published in 2006 on non-linear evolution of ballooning modes

JOREK 1.0 : Equations

- Momentum : $$\rho\frac{\partial\vec{v}}{\partial t}=-\rho\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}-\vec{\nabla}\left(\rho T\right)+\vec{J}\times\vec{B}+\mu\Delta\vec{v}$$
- Magnetic field: $$\frac{\partial\vec{A}}{\partial t}=-\eta\vec{J}+\vec{v}\times\vec{B}$$
- Continuity: $$\frac{\partial\rho}{\partial t}=-\nabla\cdot\left(\rho\vec{v}\right)+\nabla\cdot\left(D_{\perp}\nabla_{\perp}\rho+D_{\parallel}\nabla_{\parallel}\rho\right)+S$$
- Temperature: $$\frac{\partial T}{\partial t}=-v\cdot\nabla T-(\gamma-1)T\nabla\cdot v+\nabla\cdot\left(\mathrm{K}_{\perp}\nabla_{\perp}T+\mathrm{K}_{\parallel}\nabla_{\parallel}T\right)+S_T$$
- Reduced MHD : $$\vec{B}=R_0B_0\vec{a}^3+R_0\vec{\nabla}\psi\times\vec{a}^3 \qquad \vec{v}=\frac{-R^2}{R_0B_0}\vec{\nabla}u\times\vec{a}^3$$

$$\frac{\partial\psi}{\partial t}=(1+\varepsilon x)\left[\psi,u\right]+\eta\Delta^*\psi-\varepsilon\frac{\partial u}{\partial\varphi}$$

$$\frac{\partial w}{\partial t}=2\varepsilon\frac{\partial u}{\partial y}w+(1+\varepsilon x)\left[w,u\right]+\frac{1}{(1+\varepsilon x)}\left[\psi,J\right]-\frac{\varepsilon}{(1+\varepsilon x)^2}\frac{\partial J}{\partial\varphi}+\nu\nabla_{\perp}^2w$$

$$\frac{\partial\rho}{\partial t}=\ldots; \qquad \frac{\partial T}{\partial t}=\ldots$$

$$J=\Delta^*\psi; \qquad w=\nabla\cdot\nabla_{\perp}u$$

$$\psi=\tilde{\psi}aB_0$$
$$J=\tilde{J}B_0/a$$
$$u=\tilde{u}aB_0^2$$
$$w=\tilde{w}B_0^2/a$$
$$t=\tilde{t}a/B_0$$
$$\eta=\tilde{\eta}aB_0$$

JOREK (1.0) weak form

- Reduced MHD in toroidal geometry, weak form:

$$\vec{v}_\perp = \frac{-R^2}{R_0 B_0} \nabla u \times \nabla \varphi \qquad B = R_0 B_0 \nabla \varphi + R_0 \nabla \psi \times \nabla \varphi \qquad J = \Delta^* \psi \qquad w = \nabla_\perp^2 u$$

Induction equation:

$$\int \psi^* \frac{R_0}{R^2} \frac{\partial \psi}{\partial t} dV = -\int \nabla \left(\eta(T) \psi^* \right) \cdot \left(\frac{R_0}{R^2} \nabla_\perp \psi \right) dV - \int \psi^* \frac{1}{R B_0} [u, \psi] dV - \int \psi^* \frac{1}{R^2} \frac{\partial u}{\partial \phi} dV$$

Momentum equation:

$$-\frac{1}{R_0 B_0} \int \nabla u^* \cdot \left(\hat{\rho} \frac{\partial \nabla_\perp u}{\partial t} \right) dV = \int -\frac{1}{R} \left[u^*, \hat{\rho} \right] \tfrac{1}{2} v^2 dV - \hat{\rho} \frac{R}{R_0^2 B_0^2} \left[u^*, u \right] w dV + u^* \frac{R_0}{R} [\psi, j] dV$$

$$\int -u^* \frac{R_0 B_0}{R^2} \partial_3 j dV + R \left[u^*, p \right] dV - \nu(T) \nabla_\perp u^* \cdot \nabla_\perp w dV$$

Continuity equation:

$$\int \rho^* \frac{\partial \rho}{\partial t} dV = \int + \rho^* \rho \frac{2}{R_0 B_0} \partial_2 u + \rho^* \frac{R}{R_0 B_0} [\rho, u] - D_\perp \nabla \rho^* \cdot \nabla_\perp \rho + \rho^* S_\rho dV$$

Temperature equation:

$$\int T^* \frac{\partial T}{\partial t} dV = \int \left(+ \frac{2(\gamma - 1)}{R_0 B_0} T^* T \partial_2 u + \frac{R}{R_0 B_0} T^* [T, u] - \nabla T^* \cdot \left(\mathrm{K}_\perp \nabla_\perp T + \mathrm{K}_\parallel \nabla_\parallel T \right) + T^* S_T \right) dV$$

JOREK 1.0 : time evolution

- Fully implicit time evolution allows large time steps

- Linearised Crank Nicholson scheme:

$$\frac{\partial A(\vec{y})}{\partial t} = B(\vec{y})$$

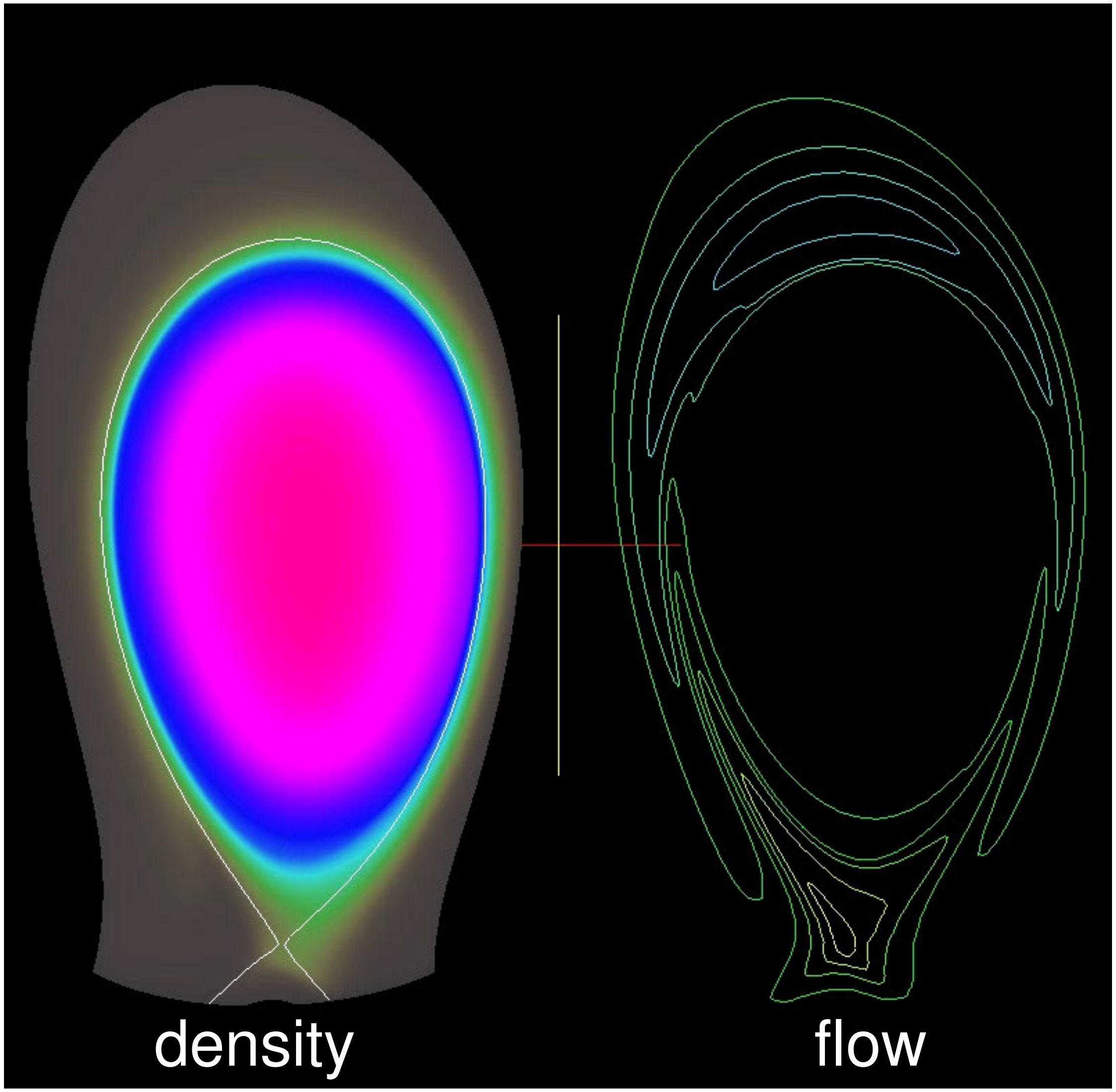
$$\frac{\partial A}{\partial y}\delta\vec{y} = \delta t B(\vec{y}_n) + \tfrac{1}{2}\delta t \frac{\partial B(\vec{y}_n)}{\partial y}\delta\vec{y}$$

$$\left(\frac{\partial A(\vec{y}_n)}{\partial y} - \tfrac{1}{2}\delta t \frac{\partial B(\vec{y}_n)}{\partial y}\right)\delta\vec{y} = B(\vec{y}_n)\delta t$$

- Large sparse system of equation solved using parallel direct sparse matrix libraries (PASTIX, MUMPS, WSMP)

Non-linear Ballooning modes

- First results, evolving n=0 and n=6 toroidal harmonics only.
- Formation of density filaments



Computing Requirements

- Present Status:
  - Evolving n=0 and n=6 toroidal harmonics
    - requires 16-32 CPU, ~64 GB memory
    - CPU time typically 2-4 days per simulation
    - mostly running locally on opteron cluster
- Eventually:
  - Evolving n=0 to 64 (256) toroidal harmonics
    - Requires (at least) 1024 CPU, 4TB memory
    - JOREK improvement necessary for better parallel scaling
      - first updated version ready for testing
- ITER challenge:
  - Lower Reynolds numbers require increased resolutions
    - 2 orders of magnitude lower compared to typical present day simulations

WP 2.2 : Development of the JOREK MHD simulation code

- Work underway
  - Bezier cubic Hermite finite elements implemented and tested (O. Czarny)
    - Reduced MHD equations as in JOREK-1.0 (no stabilisation)
    - Refinable (optional adaptive)

  - Changing time evolution scheme to less fully implicit (O. Czarny)
    - Linear terms implicit, non-linear toroidal coupling explicit (Adams-Bashfort)
      - leads to a sparse matrix for each toroidal harmonic
      - better scaling parallelisation, hopefully not losing too much on timestep
      - request for PastiX
      - Alternative : use indirect PastiX on full matrix using as preconditioner the N decoupled matrices
    - As in SFELES fluid code
    - First tests ongoing

WP 2.2 : Improvements to JOREK

- full compressible MHD model
  - Vector potential formulation to avoid div.B=0 condition
    - as used in linear MHD stability codes
  - (**V**,**A**,ρ,T) : 8 variables
- stabilised finite elements (distributed residuals)

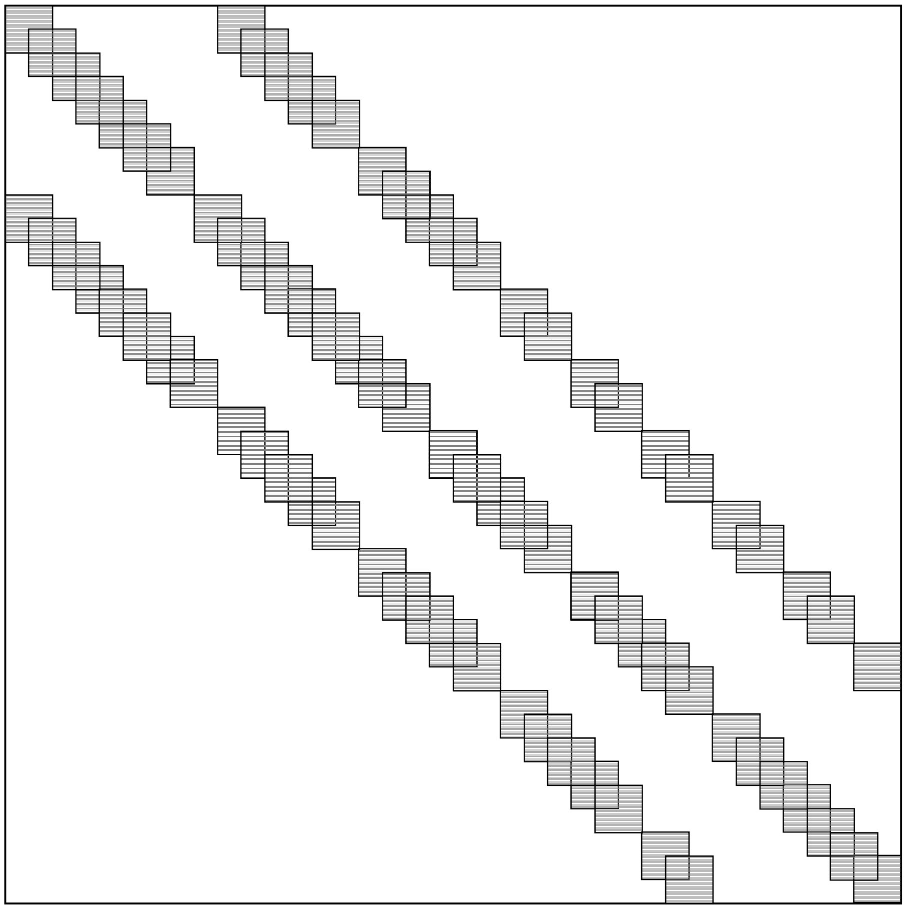
- Adaptive mesh refinement

WP 2.2 Planning

- JOREK 2.0:
  - New grid generation including x-point using Bezier elements
    - March 2007
  - Reduced MHD equations
    - April 2007
  - New time stepping scheme
    - August 2007
  - Full MHD
    - December 2007
  - Stabilised FEM
    - …
  - AMR
    - …

Matrix structure

- Current structure, one block per node

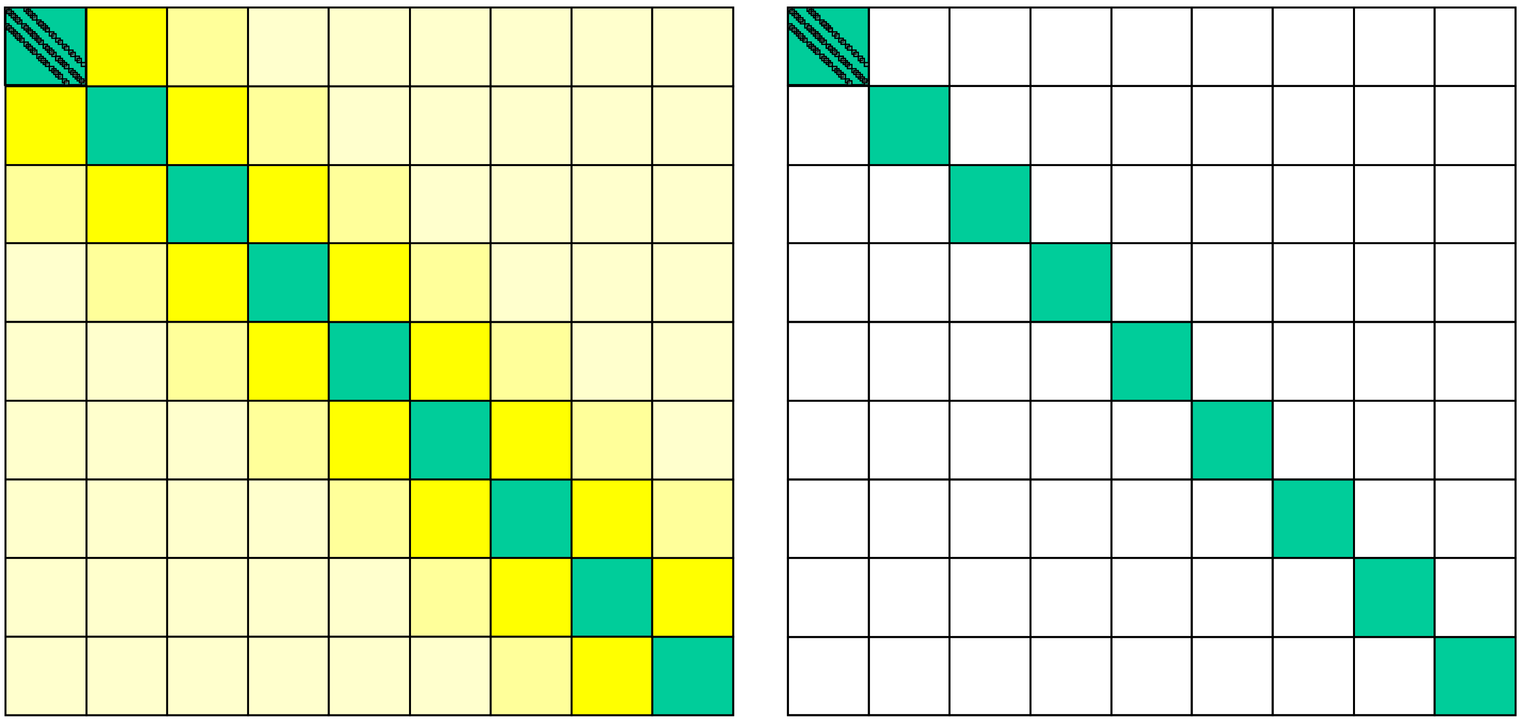


Blocksize: N(var)*N(harmonics)

Matrices

- Alternative: Each colored block has the same sparse structure
  - Block size N(var)

new scheme

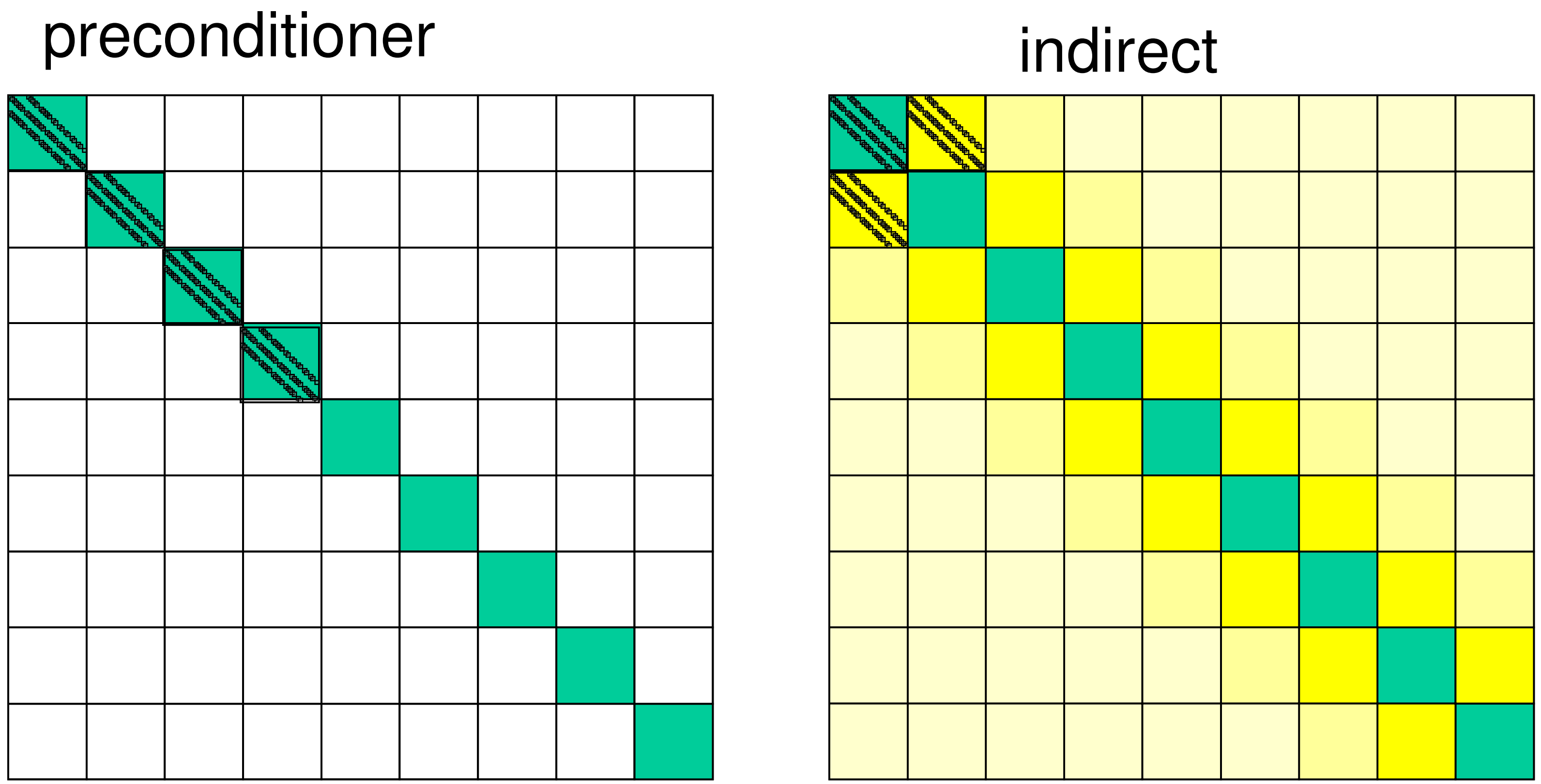


Can this be solved as one matrix in PastiX?

or solve as N independent matrices (see MUMPS)

Fully implicit direct/indirect

- Could this be an option in PastiX?



WP 4

- Application of the methods developed in the project to advance the simulation of ELMs in ITER plasmas
  - Thesis candidate to be confirmed
  - Continue with existing JOREK